SUGAR HOLDUP IN A DIFFUSER

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Abstract

Careful control of water levels in the crushed cane in a sugar diffuser is required for the efficient extraction of sucrose from the cane. The levels are monitored by observing the water surface through windows inserted in the sides of the diffuser. It is not clear,however, if these observations close to the wall reflect levels within the body of the diffuser basically because there will be a gap between the crushed cane and the wall which will effect the water flow. This is a seepage face problem which is difficult however the simple models developed here suggest that observations will be reliable.

1 Introduction

Diffusers remove sucrose from crushed and shredded cane by spraying water onto the cane and collecting the sucrose in the water mixture that drains through the cane, see Figure 1. This mixture is then processed to produce crystalline sugar. A long conveyor carries the fibre through the diffuser and a counter-current washing action is used to ensure that concentration differences between the cane and the percolating water remain relatively large. Since any sprayed water needs to be eventually removed it is important to use as little water as possible whilst still ensuring good sucrose extraction; these are conflicting objectives. In order to efficiently remove sugar from the crushed and shredded

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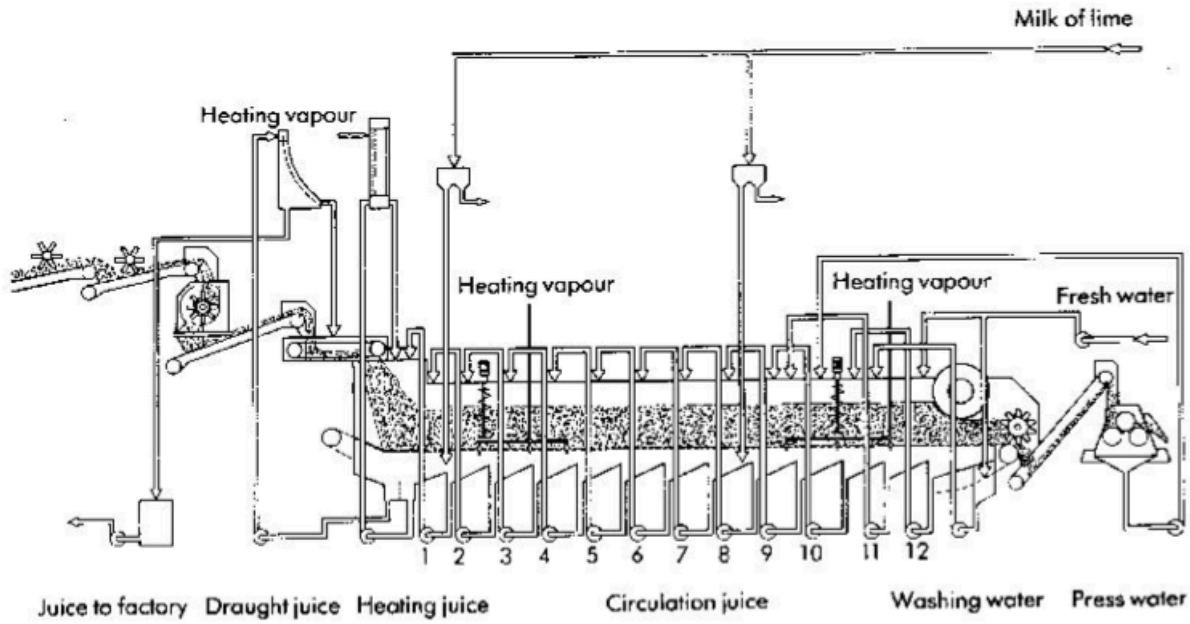

Figure 1: The Diffuser: Cane is carried into the diffuser on a belt (left hand end in the figure) and sprayed with water; a counter-current washing action is used. The water percolates through the cane and is collected in trays under the diffuser.

cane in the diffuser it is necessary to maintain water levels at the top of the cane pile in the diffuser. If this level is exceeded then flooding occurs, so that water streams from different sprays will mix and the reverse flow diffuser's purpose is defeated. If, on the other hand, water levels are lower than the top of the pile then sugar will not be efficiently removed from shredded cane close to the pile's top. Maintaining the water level at the top of the cane pile is however not an easy task because the permeability of the cane varies significantly depending on many factors (cane type, water content, fibre length and composition, density, etc.), and so varies with the feed into the diffuser. Also if flooding occurs then it occurs rapidly; decisions concerning spray volume rates need to be made online and quickly.

To facilitate the process viewing windows are inserted in the tanks so that, based on observations of the water surface as seen in these windows, operators can adjust the flow rates from the sprays. There is a gap between the shredded cane within the diffuser and the walls of the diffuser including the window, so it is not clear if water levels as seen in the windows match those within the body of the tank, and if not what corrections are needed to take into account the difference. Additionally the presence of local permeability fluctuations can result is an incorrect assessment of the water level in the body of the tank. Understanding these issues was the problem the Sugar Milling Research Institute asked the MISGSA to address.

Initially a hydrological model with gravitational flow into the air gap between the shredded cane and the window was investigated, however the results conflicted with observations. The problem appeared to be that the diffusive resistance to water movement through the shredded cane and into the gap was much greater than one might expect, so that a more detailed seepage face flow model was indicated, see Section 2. The resultant problem is however difficult and more detailed and complicated than needed for this work, so simpler models were investigated, see Section 3. It is hoped that these compromise models provide some insight concerning these issues.

2 A seepage face analysis

It has been observed in the laboratory that water does not quickly drain out of a fully exposed face of crushed cane even if the cane is saturated with water. Evidently the resistance to flow through the cane is relatively large, so that it will take some time for the water to reach the exposed face before streaming out. This may mean that there will be significant differences in water levels within the diffuser and the observation window at the side of the diffuser; we examine this situation.

The water flow rate from the sprinklers is designed to be such that there is almost a balance between sprinkler inflow and water drainage outflow, and in fact on average these flows must balance. The average drainage velocity through the cane is given by its average permeability k. If this supply rate is exceeded locally then there will be an accumulation of fluid within the pile and thus an increase in fluid height. Of course any volume element of cane will be periodically over-flushed and under-flushed as it passes under each sprinkler in turn, so that the through flow for the patch at any time will be either too much or too small especially for volume elements close to the top of the diffuser and this will lead to variations in the fluid height especially near to the diffuser walls.

We are dealing here with what is technically know as "seepage face flow", which has been much studied in a groundwater flow context, see Harr [2]. When the flow through an aquifer reaches an exposed face the water streams down the surface. The presence of this exposed face not only has local consequences but effects the flow throughout the aquefer. In the groundwater flow case the flow is quasi-steady whereas in the present case the flow is periodic so that transients play an important role. Such problems are notoriously difficult to solve because the location of the water surface, and in particular the location of the exit point for flow, needs to determined as part of the solution process. It is also questionable if unsaturated flow considerations need to be taken into account (especially in periodic flow situations), in which case the full non-linear unsaturated flow Richardson model would be required, see [1]. However it is generally assumed that such effects are small, in which case hodograph methods can be used to extract results [4]. Even so this requires sophisticated numerics and remains a daunting task that could not be addressed at the MISG. Simpler models were indicated and now will be described.

3 Simple free surface model calculations

The vertical draining flow in the diffuser is modelled in detail in the paper by Hocking and Mitchell [5] and in earlier study group reports [6]. In these papers simulations are presented for the water height within the crushed cane as observed from a fixed location outside the diffuser as the conveyor belt moves forwards with the feed. At any location along the diffuser a steady state is predicted with the depth of water in the pulp remaining fixed, even though the fluid is draining through it. Under such circumstances, after adjusting for longitudinal depth variations due to sprinkler spacing, observations through the windows would be reliable. However these simulations assumed no variations in flow across the diffuser.

As indicated earlier however it would seem very likely that there would be a small air gap between the pulp and the side of the diffuser and the observation window and, because the flow in this gap is unrestricted by the presence of cane, this could significantly effect the depth of flow at the observation window.

We consider the effect of cross diffuser permeability variations on the flow through the pile, see Figure 2. The initial height of the water column is prescribed (at H) and subsequently the water drains out. We define a Cartesian coordinate system with the origin at the base of the side wall: x measures distance across the diffuser of width $2L$, y is the vertical coordinate, and z is aligned along the length of the diffuser. Assuming a porous medium flow model, the appropriate variable is the piezometric head, $\phi(x, y, z, t)$, defined as

$$\phi = \frac{p}{\rho g} + y \tag{1}$$

where p is the pressure, g is gravity, ρ is the density of the fluid and y is the elevation. A Darcy's Law model is assumed with the flow given by

$$\mathbf{q} = -\kappa \nabla \phi. \tag{2}$$

where κ is the permeability. Combining Darcy's Law with the continuity condition ($\nabla \cdot \mathbf{q} = 0$), and assuming the porous region is homogeneous and isotropic gives Laplace's equation for ϕ. Since the fluid movement at any point moving along is vertical we ignore the variation in the z direction.

The pressure on the exposed boundaries of the region must be atmospheric, while assuming symmetry around the centre-line of the diffuser gives the no-flow condition $\phi_x = 0$ at $x = L$. Thus the full problem, with boundary conditions is given by:

$$\nabla^2 \phi = 0, \quad 0 < x < L, 0 < y < \eta(x,t), t \geq 0, \tag{3}$$

$$\phi = y \text{ on } x = 0, 0 < y < \eta(0,t), \tag{4}$$

$$\phi_x = 0 \text{ on } x = L, 0 < y < \eta(L,t), \tag{5}$$

$$\phi = 0 \text{ on } 0 < x < L, y = 0, \tag{6}$$

$$\phi = \eta \text{ on } 0 < x < L, y = \eta(x,t), \tag{7}$$

$$\eta_t + \kappa \phi_x \eta_x + \kappa \phi_y = 0 \text{ on } y = \eta(x,t), \tag{8}$$

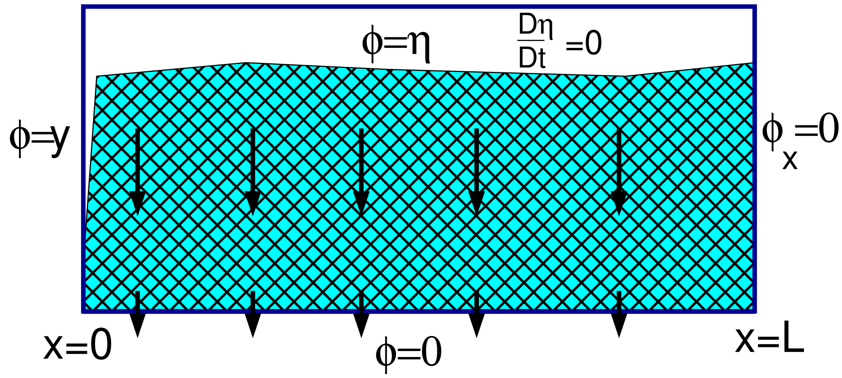

Figure 2: Sketch of the diffuser bed of width $2L$. The conveyor is moving toward the reader. At an instant frozen in time the level of water in the bed is constant at this location. $\frac{D\eta}{Dt} = 0$ represents equation (8).

where $y = \eta(x,t)$ is the equation of the surface of the liquid with initial height $\eta(x,0) = H$ prescribed. The final two conditions (7, 8) are applied on the free surface of the liquid in the bed. The first ensures continuity of pressure while the second states that fluid particles move with the surface.

As pointed out earlier this system is complicated, requiring as it does a determination of the variable location of the surface including the exit point from the pile. However there is a simple exact solution given by

$$\phi(x,y) = y, \text{ with } \eta(x,t) = H - \kappa t, \tag{9}$$

where H is the height of the water surface associated with the a fixed influx into and through the bed. This flow corresponds to "natural" steady drainage of speed κ through the pile. For this ideal solution there is no flow through the seepage face; in essence the air gap is not there. In such circumstances the fluid height will be uniform across the diffuser and so the level shown in the side windows will accurately reflect the level inside the diffuser.

3.1 Permeability variations

Under normal operating conditions, however, there will be higher and lower permeability regions in the bed of the diffuser. Such variations are probably not predictable so that the associated differential height rise is not predictable. However the effect of such variations would be to cause an excess or deficit of flow through the system and, because the steady drainage flow is natural for the system, one might expect this would result in the excess fluid spilling out through the seepage face, and any deficit would result in an increased gap size as continuity would dictate.

Based on these ideas we consider the effect of modifying the surface flux conditions (7, 8). We replace these conditions by an imposed flux condition at H. Explicitly we assume

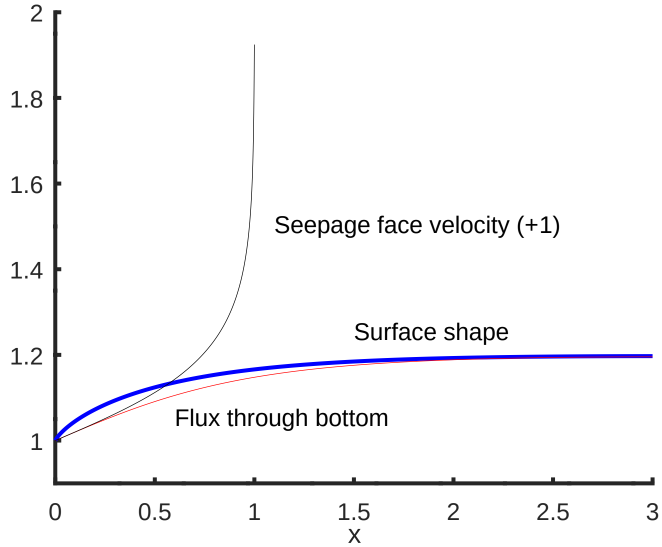

Figure 3: Liquid surface shape and velocity through the seepage face and through the bottom of the diffuser bed for $\kappa = 1$, $L/H = 3$. Note the seepage face velocity is offset by one unit to simplify the axis scale.

the flux is given by

$$\phi_y(x, H) = 1 + \varepsilon. \tag{10}$$

where ε represents a modification to the "ideal" flow condition; $\phi_y = 1$ is the influx associated with the ideal exact solution. Thus if ε is positive, then there is a slight excess of inflow, while if ε is negative, there is a deficit.

This modified problem can be solved using Fourier series after first extracting the ideal solution and is given by

$$\phi(x, y) = y/H + \frac{2\varepsilon H}{L} \sum_{k=0}^{\infty} \frac{\sinh(\lambda_k y/H)}{\lambda_k^2 \cosh(\lambda_k)} \sin(\lambda_k x/H), \tag{11}$$

where

$$\lambda_k = \frac{(2k+1)\pi}{2L}, k = 0, 1, 2, \ldots \tag{12}$$

are the appropriate eigenvalues. An approximation for the phreatic surface about $y = H$ is given by lines of constant head, and that nearest to $y = H$ is

$$\eta(x) \approx \phi(x, H) = 1 + \frac{2\varepsilon H}{\bar{L}} \sum_{k=0}^{\infty} \frac{\tanh \lambda_k \sin(\lambda_k x/H)}{\lambda_k^2} \tag{13}$$

Figure 3 shows the resulting (approximate) shape of the free surface for a case of $\varepsilon = 0.1$. It also shows that the flux through the base is slightly increased, but is lower near the seepage face. The velocity through the seepage face is zero near the base, and there is a singularity at the point where the surface meets the seepage face. Overall the total volume flowing out through the seepage face is quite small. The surface dips down at the outer edge as there is some flow out through the seepage face. This is perhaps counter-intuitive as more liquid is being forced in from the top, but this is being compensated for by the flow out through the seepage face. There is also a slight increase in flow out through the base.

This might model a case where there is a high permeability region in the bed, so that rather than water being forced in from above, there are less pathways to remove the existing liquid, leaving an excess. In the opposite case, where $\varepsilon < 0$ there is slight rise in the surface. This is problematic because mathematically this is due to a slight “suction” on the vertical face. This would result in a drying of this face and hence would act as a barrier to down-flow, causing a slight build up at the top of the bed.

The implication is that if there is some hold up in the flow, then the level in the viewing window would rise, while if the flow is going faster, due perhaps to a higher permeability region, then it would drop. It is important to note that these conclusions are based on a very small perturbation to the exact natural drainage solution, but typically such approximations are quite good. These conclusions are based on the assumption that the water level in the window is a reflection of the actual water level in the diffuser bed, rather than the water level in the gap between the moving bed and the side walls. From these results a 5% disruption in the downflow would result in a roughly similar movement up or down at the window level, depending on the width of the bed. However, the disruption occurs near the edge and so the flow in the middle may actually not be affected much at all.

3.2 Wall flow effects

Given the flow is strongly vertical in the ideal situation, we can modify the equations to take into account the fact that the permeability may be greater in a small region near to the diffuser wall (at $x = 0$) due to the movement along the channel. Explicitly we model this variation as

$$\kappa(x) = \kappa_0 + \varepsilon g(x) \;, \tag{14}$$

where ε is a small parameter. Under these circumstances the equation of flow becomes

$$\frac{\partial}{\partial x}\left(\kappa(x)\frac{\partial \phi}{\partial x}\right) + \kappa(x)\frac{\partial^2 \phi}{\partial y^2} = 0 \tag{15}$$

and the same boundary conditions as before apply.

We seek a solution of the form

$$\phi(x, y, t) = \phi_0 + \varepsilon\phi_1 + \ldots, \qquad \text{with} \qquad \eta(\mathrm{x}, \mathrm{t}) = \eta_0 + \varepsilon\eta_1 + \ldots, \tag{16}$$

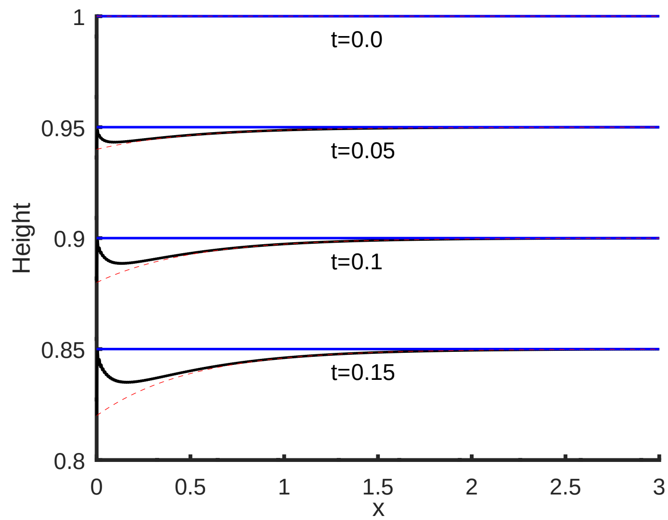

Figure 4: Liquid surface shape and velocity through the seepage face and through the bottom of the diffuser bed for $\kappa_0 = 1$, $\varepsilon = 0.2$, $\alpha = 2$, $L = 3$. The blue line indicates the leading order draining solution and the black includes the $\mathcal{O}(\varepsilon)$ term. The dashed line indicates pure draining with no edge effects.

and substituting into (15) leads to the following problems;

$$\mathcal{O}(1): \quad \nabla^2\phi_0 = 0 \tag{17}$$

$$\eta_{0t} - \kappa_0\phi_{0x}\eta_{0x} + \kappa_0\phi_{0y} = 0, \quad \text{on } y = \eta_0(x,t) \tag{18}$$

$$\phi_0 = \eta_0, \quad \text{on } y = \eta_0(x,t) \tag{19}$$

$$\phi_0 = y \text{ on } x = 0, \tag{20}$$

$$\phi_{0x} = 0 \text{ on } x = L, \tag{21}$$

$$\phi = 0 \text{ on } y = 0, \tag{22}$$

for which the solution is (9), with $\phi_0 = y$, where H is the initial level.

At the next level,

$$\mathcal{O}(\varepsilon): \quad g(x)\nabla^2\phi_0 + \kappa_0\nabla^2\phi_1 + g(x)\phi_{1x} = 0 \tag{23}$$

$$\eta_{1t} - \kappa_0\phi_{0x}\eta_{1x} - \kappa_0\phi_{1x}\eta_{0x} - g(x)\phi_{0x}\eta_{0x} \tag{24}$$

$$+ \kappa_0\phi_{1y} + g(x)\phi_{0y} = 0, \quad \text{on } y = \eta_0(x,t) \tag{25}$$

$$\phi_1 = \eta_1, \quad \text{on } y = \eta_0(x,t) \tag{26}$$

$$\phi_1 = 0 \text{ on } x = 0, \tag{27}$$

$$\phi_{1x} = 0 \text{ on } x = L, \tag{28}$$

$$\phi_1 = 0 \text{ on } y = 0. \tag{29}$$

However, many of these terms are zero, so that the problem becomes

$$\mathcal{O}(\varepsilon): \nabla^2\phi_1 = 0 \tag{30}$$

$$\eta_{1t} + \kappa_0\phi_{1y} + g(x) = 0, \quad \text{on } y = H - \kappa_0 t \tag{31}$$

$$\phi_1 = \eta_1, \quad \text{on } y = H - \kappa_0 t \tag{32}$$

$$\phi_1 = 0 \text{ on } x = 0, \tag{33}$$

$$\phi_{1x} = 0 \text{ on } x = L, \tag{34}$$

$$\phi_1 = 0 \text{ on } y = 0. \tag{35}$$

This can be solved as a time dependent Fourier series problem by letting

$$\phi_1(x, y, t) = \sum_{k=0}^{\infty} a_k(t)\sinh\lambda_k y \sin\lambda_k x, \tag{36}$$

$$\eta_1(x, t) = \sum_{k=0}^{\infty} b_k(t)\sin\lambda_k x, \tag{37}$$

where

$$\lambda_k = \frac{(2k+1)\pi}{2L}, \qquad k = 0, 1, 2, \ldots \tag{38}$$

ensures conditions are satisfied on $x = 0, L$ and $y = 0$.

Now, on $y = H - \kappa_0 t$, $\phi_1 = \eta_1$, so

$$b_k(t) = a_k(t)\sinh\lambda_k(H - \kappa_0 t), \tag{39}$$

and substituting into the final surface condition,

$$\sum_{k=0}^{\infty}(b_k'(t) + \kappa_0 a_k(t)\cosh\lambda_k(H - \kappa_0 t))\sin\lambda_k x = -g(x) \tag{40}$$

and so we can write

$$b_k'(t) + \kappa_0\lambda_k\coth\lambda_k(H - \kappa_0 t)b_k(t) = -d_k, \;\; k = 0, 1, 2, \ldots \tag{41}$$

where

$$\sum_{k=0}^{\infty} d_k\sin\lambda_k x = -g(x), \tag{42}$$

so that

$$d_k = -\frac{2}{L}\int_0^L g(x)\sin\lambda_k x \; dx, \tag{43}$$

providing a set of differential equations for $b_k(t)$, $k = 0, 1, 2, \ldots$. An initally flat liquid surface gives $b_k(0) = a_k(0) = 0$, $k = 0, 1, 2, \ldots$ While in general this system cannot be solved exactly, it can be solved very easily using a simple numerical scheme.

The instantaneous flux through the seepage face can be obtained from

$$Q_S(t) = \kappa_0 \int_0^{H-\kappa_0 t} \phi_{1x}(0, y, t)dy = \kappa_0 \sum_{k=0}^{\infty} a_k(t)\lambda_k \int_0^{H-\kappa_0 t} \sinh \lambda_k y \; dy$$
$$= \kappa_0 \sum_{k=0}^{\infty} a_k(t) \left[\cosh \lambda_k (H - \kappa_0 t) - 1\right] \; . \tag{44}$$

Consider a simple example in which $g(x) = e^{-\alpha x}$, then equation (43) becomes

$$d_k = \frac{2(\lambda_k - \alpha e^{-\alpha x} \sin(\lambda_k L))}{L(\alpha^2 + \lambda_k^2)} = \frac{2(\lambda_k - (-1)^k \alpha e^{-\alpha x})}{L(\alpha^2 + \lambda_k^2)} \tag{45}$$

A series of surface solutions at different times for $g(x) = \exp(-\alpha x)$ is shown in Figure 4, for $\varepsilon = 0.2$, $L = 3$ and $\alpha = 2$. Different times can also be thought of as different distance down the diffuser from the inlet sprays, as shown in [5]. The dashed line indicates the "pure draining" solution that would occur if the edge effects were not included. The rise at the edge in the solution is an effect of the atmospheric pressure acting on the side of the diffuser bed. This is slightly surprising and raises the surface level back up to close to where it would be for isotropic flow. Furthermore, calculations indicate that the pressure gradient at the edge of the bed would be such that there would be almost no flow out through the seepage face. However, if there was a region of decreased permeability in this system, a small flow out through the seepage face would result as the liquid surface would slope slightly downward at the outer edge.

3.3 Comments

The calculations in this section suggest that the level of water in the glass should generally remain quite stable, and this should certainly be so in "ideal", equilibrium operating conditions. However, small increases or decreases in the permeability can lead not only to small up or down variations (depending on whether it is a decrease or increase in permeability) but to flow through the seepage face which might lead to sudden fluctuations, or in the case of a higher internal permeability a drying of the bagasse at the edge of the diffuser. Importantly it seems that due to conservation of mass the changes in level due to these different disruptions are manifested near the edge of the diffuser, where the viewing windows occur, so that variations in the level there may not be a reflection of changes across the whole diffuser.

The change in level seen appears to be of the order of the disruption from the equilibrium flow situation, although if there is flow out through the seepage face one might expect a flooding of the window, but this may not be indicative of flooding of the diffuser, as a sudden decrease in permeability near to the edge may result in a flow through the seepage face.

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